Title: Springs and Numerical Analysis

Brief Overview:

Students will experimentally determine the force law for springs (Hooke's Law). Using the basic definitions of velocity and acceleration they will write equations for the velocity and distance a fraction of a second later, then write a calculator program which will loop through the equations to generate a time series of distance, velocity, and acceleration. Graphs will show the relationship between the three variables. The use of numerical analysis eliminates the need for calculus to arrive at the sinusoidal functions. The unit ends with a discussion of connections to more advanced problem solving using numerical analysis.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*

- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*

- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*

- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student's learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Algebra
Difference equations must be rearranged for iterative computation.

Measurement
Measurements of force, distance, and time must fit together to agree with the numerical predictions of the spring motion.

Data Analysis and Probability
Students will not be directed to the fact that part of the spring mass is oscillating until they have shown that discrepancies between data and predictions can not be explained by measurement errors.

- **Process Standards**

  <u>Problem Solving</u>
  Stepwise links between the variables used in iteration must be worked out by the students after being modeled by the teacher.

  <u>Representation</u>
  Graphs of distance and velocity as a function of time are generated from the numerical data.

Links to National Science Education Standards

- **Science as Inquiry**
  Are the simple laws of motion really sufficient to describe the complex oscillation of a spring?

- **Physical Science**
  Students will investigate Newton’s laws and kinematic equations.

- **Science and Technology**
  The use of calculators for numerical integration is a major theme in modern design.

Links to Maryland High School Science Core Learning Goals

- **Concepts of Physics**
  Students will know and apply the laws of mechanics to explain the behavior of the physical world.

Grade/Level:

Grades 10-12, Honors or Gifted and Talented

Duration:

1.5 hours

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Use of velocity and acceleration definitions as algebraic equations
- Understanding of Newton’s second law
- Calculating weight as a force from mass
- Use of the TI-83 calculator graphing and statistical functions

Student Outcomes:

Students will:

- derive the force equation for springs from experimental data.
- rearrange definitions of velocity and acceleration to find values of distance and velocity an incremental time later.
- program the TI-83 calculator to perform iterative calculations.

Materials/Resources/Printed Materials:

- Springs with method to suspend them
- Weight sets
- Stopwatches
- TI programmable calculators
- “Motion by Increments” chart if calculators are unavailable
- CBL motion detector (optional)

Development/Procedures:

- **Motivation:** ‘How does the force produced by a spring depend on the extension? Can anyone explain how this force would produce the motion we see when a mass is pulled back and released? We will connect these two observations by a powerful yet simple technique known as “numerical analysis”. You may have heard this described as “computer simulation”.’

- **Spring Force:** Instruct the students to measure spring extension versus force for at least 5 masses (converted to weight) and draw a graph of force as a function of extension. They must find the slope and write an equation for force as a function of extension using units of Newtons and Meters.

  -> **Formative Assessment:** Did the students calculate the slope correctly?

- **Discussion:** What type of proportionality is $F = -kx$? What is the meaning of the negative sign? (they are not likely to have put this in themselves). What happens to the “spring constant” as the spring becomes stiffer?

- **Oscillatory Motion:** The easiest part of the motion to measure is the period of oscillation. With a small mass the spring may bounce rather quickly; ask students what they can do to measure the period of a single bounce accurately. Have them measure the period for at least 3 different masses.

- **Numerical Analysis:** We can use Newton’s Laws to predict the acceleration from the force, then predict velocity a moment later from the acceleration, as well as position from the velocity. (It may be necessary to point out that at equilibrium the spring force balances out gravity, so if all distances are measured from this equilibrium point, we can ignore gravity as canceled out.)

  To do this we must start with our initial conditions x_o and v_o, then use our knowledge of motion to calculate these values (x_1 and v_1) a short time Dt later:

$$a_1 = - k\, x_o/m$$
$$v_1 = v_o + a_o\Delta t$$
$$x_1 = x_o + v_1\Delta t$$

  or more precisely $x_1 = x_o + 1/2(v_o+v_1)Dt$ where the average velocity is used to find the new value of x.

This procedure can be iterated to find each successive value of position and velocity. A sample calculator program which inputs the initial conditions, iterates, stores, and graphs the values of position and velocity versus time is included as a sample page. If the calculators are not available or the instructor is not familiar with the programming capability, the same process can be carried out by hand in a spreadsheet format which is included.

-> **Formative Assessment:** How do the values for period you get from your calculator model compare to those you measured earlier today? Input the same values for k and m.

- **Discussion:** do the values match? If not, are they within experimental error? (If the students are really careful, small mass measurements will not match). What could we be leaving out? (Answer: the spring has mass, and is oscillating as well! Try to add 1/3 the mass of the spring to the experimental mass, and see if it agrees with experiment).

Assuming the values can be made to match, what use is this technique? If you needed to predict the motion of something very complex and expensive (car shock absorber system) would it be cheaper to experiment with the actual device, or cheaper to write a computer model?

Assessment:

Formative assessments have been suggested throughout the lesson. The following summative assessments could test deep understanding.

- The three experimental masses each yielded a period. Given a fourth mass in between the largest two masses, predict the period by graphical means. Verify it by your calculator model and by direct experiment. Turn in all three values to the teacher with a description of how you obtained them, and an explanation of any differences among the values.

- The program can be modified to predict the motion of any object if given the force. If the force of gravity on the moon is 1/6 that of the earth, predict how far an object would fall in one second and how fast it would hit the ground after that one second.

Extension/Follow Up:

The assessments provide great extensions, but you may ask your class about other uses for computer modeling they may have heard of. Examples are calculations of ballistic motion with air resistance, climate modeling, etc. Have them write an investigative report on what information goes into building each of these models.

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